Math 201

An overview of the exam problems.
Take a minute to look at all the questions, THEN
solve each problem on its corresponding page INSIDE the booklet.

1. (5 pts each part, 30 pts total) For each of the following series, determine whether it converges or diverges. Justify your answer.

a) $\sum_{n=0}^{\infty} \frac{n+1}{n+2}$ b) $\sum_{n=0}^{\infty} \frac{n!(2n)!}{(3n)!}$ c) $\sum_{n=2}^{\infty} \left(1-\frac{6}{n}\right)^{n^2}$

d) $\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n}}$ e) $\sum_{n=1}^{\infty} \frac{n^2 \sin n}{2^n}$ f) $\sum_{n=1}^{\infty} \frac{\sin(1/n^{0.6})}{n^{0.7}}$

2. (5 pts each part, 10 pts total)

a) Show that the following series converges, and find its value (you are **not** required to simplify the expression):

$$\sum_{n=1}^{\infty} \left(\frac{(-5)^{n+1}}{7^n} + \frac{3^{n-1}}{4^{n+2}} \right)$$ **Careful!** The series starts at $n = 1$.

b) Find the value of the following series (you do not have to show that it converges):

$$\sum_{k=2}^{\infty} \frac{(-1)^k \pi^{2k}}{(2k)!}$$ **Careful!** The series starts at $k = 2$.

3. (15 pts) For which values of x does the following power series converge? Also, for which values of x is the convergence absolute? (Remember to test the endpoints!)

$$\sum_{n=0}^{\infty} \frac{(x-8)^n}{2^n(n+2)}.$$

4. (10 pts each part, 20 pts total)

a) Find the second-order Taylor polynomial $P_2(x)$ for the function $f(x) = \ln(3x+2)$ **centered at** $x = 1$.

(Your answer will have the form $P_2(x) = c_0 + c_1(x-1) + c_2(x-1)^2$ with specific numbers c_0, c_1, c_2 that you must find. Be careful with taking derivatives.)

b) Use Taylor's theorem to show that $|f(1.1) - P_2(1.1)| \leq 10^{-4}$.

Possibly useful numbers: $4^3 = 64$, $5^3 = 125$, $6^3 = 216$.

5. (25 pts total)

a) (9 pts) Express the following integral as a series:

$$L = \int_{x=0}^{0.1} \frac{e^{-x^2} - 1}{x} dx.$$

b) (9 pts) Find a specific partial sum s_N for which you can show that $|s_N - L| \leq 10^{-12}$.

c) (7 pts) Challenge: answer the same question as in part (b) for the different integral

$$M = \int_{x=0}^{0.1} \frac{e^{x^2} - 1}{x} dx.$$

This means that you should find a specific partial sum t_N of a different series for which you can show that that $|t_N - M| \leq 10^{-12}$.